

Measurements of the absolute branching fractions of  $D^{0(+)} \rightarrow K\bar{K}\pi\pi$  decays

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Based on  $e^+e^-$  collision data sample corresponding to an integrated luminosity of  $2.93 \text{ fb}^{-1}$  taken at a center-of-mass energy of  $3.773 \text{ GeV}$  by the BESIII detector, we report the measurements of the absolute branching fractions of  $D^0 \rightarrow K^+K^-\pi^0\pi^0$ ,  $D^0 \rightarrow K_S^0K_S^0\pi^+\pi^-$ ,  $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$ ,  $D^0 \rightarrow K_S^0K^+\pi^-\pi^0$ ,  $D^+ \rightarrow K^+K^-\pi^+\pi^0$ ,  $D^+ \rightarrow K_S^0K^+\pi^0\pi^0$ ,  $D^+ \rightarrow K_S^0K^-\pi^+\pi^+$ ,  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$ , and  $D^+ \rightarrow K_S^0K_S^0\pi^+\pi^0$ . The decays  $D^0 \rightarrow K^+K^-\pi^0\pi^0$ ,  $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$ ,  $D^0 \rightarrow K_S^0K^+\pi^-\pi^0$ ,  $D^+ \rightarrow K_S^0K_S^0\pi^+\pi^0$ , and  $D^+ \rightarrow K_S^0K^+\pi^0\pi^0$  are observed for the first time. The branching fractions of the decays  $D^0 \rightarrow K_S^0K_S^0\pi^+\pi^-$ ,  $D^+ \rightarrow K^+K^-\pi^+\pi^0$ ,  $D^+ \rightarrow K_S^0K^-\pi^+\pi^+$ , and  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$  are measured with improved precision compared to the world-average values.

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## I. INTRODUCTION

Multibody hadronic  $D^{0(+)}$  decays provide an ideal laboratory to study strong and weak interactions. Amplitude analyses of these decays offer comprehensive information of quasi-two-body  $D^{0(+)}$  decays, which are important to explore  $D\bar{D}^0$  mixing, charge conjugation-parity ( $CP$ ) violation, and quark  $SU(3)$ -flavor asymmetry breaking phenomenon [1–5]. In particular, for the search of  $CP$  violation, it is important to understand the intermediate structures for the singly Cabibbo-suppressed decays of  $D^{0(+)} \rightarrow K\bar{K}\pi\pi$  [6–8].

Current measurements of the  $D^{0(+)} \rightarrow K\bar{K}\pi\pi$  decays containing  $K_S^0$  or  $\pi^0$  are limited [9]. The branching fractions (BFs) of  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$  [10,11],  $D^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^+$  [12],  $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$  [12], and  $D^+ \rightarrow K^+ K^- \pi^+ \pi^0$  [13] were only determined relative to some well-known decays or via topological normalization, with poor precision. This paper presents the first direct measurements of the absolute BFs for the decays  $D^0 \rightarrow K^+ K^- \pi^0 \pi^0$ ,  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$ ,  $D^0 \rightarrow K_S^0 K^- \pi^+ \pi^0$ ,  $D^0 \rightarrow K_S^0 K^+ \pi^- \pi^0$ ,  $D^+ \rightarrow K^+ K^- \pi^+ \pi^0$ ,  $D^+ \rightarrow K_S^0 K^+ \pi^0 \pi^0$ ,  $D^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$ ,  $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ , and  $D^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^0$ . The  $D^0 \rightarrow K_S^0 K_S^0 \pi^0 \pi^0$  decay is not included since it suffers from poor statistics and high background. Throughout this paper, charge conjugate processes are implied. An  $e^+e^-$  collision data sample corresponding to an integrated luminosity of  $2.93 \text{ fb}^{-1}$  [14] collected at a center-of-mass energy of  $\sqrt{s} = 3.773 \text{ GeV}$  with the BESIII detector is used to perform this analysis.

## II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector is a magnetic spectrometer [15] located at the Beijing Electron Positron Collider (BEPCII) [16]. The cylindrical core of the BESIII detector consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing an 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identifier modules interleaved with steel. The acceptance of charged particles and photons is 93% over  $4\pi$  solid angle. The charged-particle momentum resolution at  $1 \text{ GeV}/c$  is 0.5%, and the  $dE/dx$  resolution is 6% for the electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution of the TOF barrel part is 68 ps, while that of the end cap part is 110 ps.

Simulated samples produced with the GEANT4-based [17] Monte Carlo (MC) package including the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the backgrounds. The simulation includes

the beam-energy spread and initial-state radiation (ISR) in the  $e^+e^-$  annihilations modeled with the generator KKMC [18]. The inclusive MC samples consist of the production of  $D\bar{D}$  pairs with consideration of quantum coherence for all neutral  $D$  modes, the non- $D\bar{D}$  decays of the  $\psi(3770)$ , the ISR production of the  $J/\psi$  and  $\psi(3686)$  states, and the continuum processes. The known decay modes are modeled with EVTGEN [19] using the BFs taken from the Particle Data Group (PDG) [9], and the remaining unknown decays from the charmonium states are modeled with LUNDCHARM [20]. The final-state radiations from charged final-state particles are incorporated with the PHOTOS package [21].

## III. MEASUREMENT METHOD

The  $D^0\bar{D}^0$  or  $D^+D^-$  pair is produced without an additional hadron in  $e^+e^-$  annihilations at  $\sqrt{s} = 3.773 \text{ GeV}$ . This process offers a clean environment to measure the BFs of the hadronic  $D$  decay with the double-tag (DT) method. The single-tag (ST) candidate events are selected by reconstructing a  $\bar{D}^0$  or  $D^-$  in the following hadronic final states:  $\bar{D}^0 \rightarrow K^+\pi^-$ ,  $K^+\pi^-\pi^0$ , and  $K^+\pi^-\pi^+\pi^-$ , and  $D^- \rightarrow K^+\pi^-\pi^-$ ,  $K_S^0\pi^-$ ,  $K^+\pi^-\pi^-\pi^0$ ,  $K_S^0\pi^-\pi^0$ ,  $K_S^0\pi^+\pi^-\pi^-$ , and  $K^+K^-\pi^-$ . The event in which a signal candidate is selected in the presence of an ST  $\bar{D}$  meson, is called a DT event. The BF of the signal decay is determined by

$$\mathcal{B}_{\text{sig}} = N_{\text{DT}}^{\text{net}} / (N_{\text{ST}}^{\text{tot}} \cdot \epsilon_{\text{sig}}), \quad (1)$$

where  $N_{\text{ST}}^{\text{tot}} = \sum_i N_{\text{ST}}^i$  and  $N_{\text{DT}}^{\text{net}}$  are the total yields of the ST and DT candidates in data, respectively.  $N_{\text{ST}}^i$  is the ST yield for the tag mode  $i$ . For the signal decays involving  $K_S^0$  meson(s) in the final states,  $N_{\text{DT}}^{\text{net}}$  is the net DT yields after removing the peaking background from the corresponding non- $K_S^0$  decays. For the other signal decays, the variable corresponds to the fitted DT yields as described later. Here,  $\epsilon_{\text{sig}}$  is the efficiency of detecting the signal  $D$  decay, averaged over the tag mode  $i$ , which is given by:

$$\epsilon_{\text{sig}} = \sum_i (N_{\text{ST}}^i \cdot \epsilon_{\text{DT}}^i / \epsilon_{\text{ST}}^i) / N_{\text{ST}}^{\text{tot}}, \quad (2)$$

where  $\epsilon_{\text{ST}}^i$  and  $\epsilon_{\text{DT}}^i$  are the efficiencies of detecting ST and DT candidates in the tag mode  $i$ , respectively. For  $D^0$  decay, the signal efficiency has been corrected by a factor considering the Cabibbo-favored and doubly Cabibbo-suppressed contributions of the tag sides, as shown in Sec. VIII.

## IV. EVENT SELECTION

The selection criteria of  $K^\pm$ ,  $\pi^\pm$ ,  $K_S^0$ , and  $\pi^0$  are the same as those used in the analyses presented in Refs. [22–31]. All charged tracks, except those from  $K_S^0$  decays, are required

to have a polar angle  $\theta$  with respect to the beam direction within the MDC acceptance  $|\cos\theta| < 0.93$ , and a distance of closest approach to the interaction point (IP) within 10 cm along the beam direction and within 1 cm in the plane transverse to the beam direction. Particle identification (PID) for charged pions, kaons, and protons is performed by exploiting TOF information and the specific ionization energy loss  $dE/dx$  measured by the MDC. The confidence levels for pion and kaon hypotheses ( $CL_\pi$  and  $CL_K$ ) are calculated. Kaon and pion candidates are required to satisfy  $CL_K > CL_\pi$  and  $CL_\pi > CL_K$ , respectively.

The  $K_S^0$  candidates are reconstructed from two oppositely charged tracks to which no PID criteria are applied and which masses are assumed to be that of pions. The charged tracks from the  $K_S^0$  candidate must satisfy  $|\cos\theta| < 0.93$ . In addition, due to the long lifetime of the  $K_S^0$  meson, there is a less stringent criterion on the distance of closest approach to the IP in the beam direction of less than 20 cm and no requirement on the distance of closest approach in the plane transverse to the beam direction. Furthermore, the  $\pi^+\pi^-$  pairs are constrained to originate from a common vertex and their invariant mass is required to be within (0.486, 0.510)  $\text{GeV}/c^2$ , which corresponds to about three times the fitted resolution around the nominal  $K_S^0$  mass. The decay length of the  $K_S^0$  candidate is required to be larger than two standard deviations of the vertex resolution away from the IP.

The  $\pi^0$  candidate is reconstructed via its  $\gamma\gamma$  decay. The photon candidates are selected using the information from the EMC shower. It is required that each EMC shower starts within 700 ns of the event start time and its energy is greater than 25 (50) MeV in the barrel (end cap) region of the EMC [15]. The energy deposited in the nearby TOF counters is included to improve the reconstruction efficiency and energy resolution. The opening angle between the candidate shower and the nearest charged track must be greater than  $10^\circ$ . The  $\gamma\gamma$  pair is taken as a  $\pi^0$  candidate if its invariant mass is within (0.115, 0.150)  $\text{GeV}/c^2$ . To improve the resolution, a kinematic fit constraining the  $\gamma\gamma$  invariant mass to the  $\pi^0$  nominal mass [9] is imposed on the selected photon pair.

## V. YIELDS OF ST $\bar{D}$ MESONS

To select  $\bar{D}^0 \rightarrow K^+\pi^-$  candidates, the backgrounds from cosmic rays and Bhabha events are rejected by using the same requirements described in Ref. [32]. In the selection of  $\bar{D}^0 \rightarrow K^+\pi^-\pi^-\pi^+$  candidates, the  $\bar{D}^0 \rightarrow K_S^0 K^\pm \pi^\mp$  decays are suppressed by requiring the mass of all  $\pi^+\pi^-$  pairs to be outside (0.478, 0.518)  $\text{GeV}/c^2$ .

The tagged  $\bar{D}$  mesons are identified using two variables, namely the energy difference

$$\Delta E_{\text{tag}} \equiv E_{\text{tag}} - E_b, \quad (3)$$

and the beam-constrained mass

$$M_{\text{BC}}^{\text{tag}} \equiv \sqrt{E_b^2 - |\vec{p}_{\text{tag}}|^2}. \quad (4)$$

Here,  $E_b$  is the beam energy,  $\vec{p}_{\text{tag}}$  and  $E_{\text{tag}}$  are the momentum and energy of the  $\bar{D}$  candidate in the rest frame of  $e^+e^-$  system, respectively. For each tag mode, if there are multiple candidates in an event, only the one with the smallest  $|\Delta E_{\text{tag}}|$  is kept. The tagged  $\bar{D}$  candidates are required to satisfy  $\Delta E_{\text{tag}} \in (-55, 40)$  MeV for the tag modes containing  $\pi^0$  in the final states and  $\Delta E_{\text{tag}} \in (-25, 25)$  MeV for the other tag modes, thereby taking into account the different resolutions.

To extract the yields of ST  $\bar{D}$  mesons for individual tag modes, binned-maximum likelihood fits are performed on the corresponding  $M_{\text{BC}}^{\text{tag}}$  distributions of the accepted ST candidates following Refs. [22–28]. In the fits, the  $\bar{D}$  signal is modeled by an MC-simulated shape convolved with a double-Gaussian function describing the resolution difference between data and MC simulation. The combinatorial background shape is described by an ARGUS function [33] defined as  $c_f(f; E_{\text{end}}, \xi_f) = A_f \cdot f \cdot \sqrt{1 - (f^2/E_{\text{end}}^2)} \cdot \exp[\xi_f(1 - f^2/E_{\text{end}}^2)]$ , where  $f$  denotes  $M_{\text{BC}}^{\text{tag}}$ ,  $E_{\text{end}}$  is an endpoint fixed at 1.8865 GeV,  $A_f$  is a normalization factor, and  $\xi_f$  is a free parameter. The resulting fits to the  $M_{\text{BC}}$  distributions for each mode are shown in Fig. 1. The total yields of the ST  $\bar{D}^0$  and  $D^-$  mesons in data are  $2327839 \pm 1860$  and  $1558159 \pm 2113$ , respectively, where the uncertainties are statistical only. For  $D^- \rightarrow K_S^0 \pi^-$ ,  $D^- \rightarrow K_S^0 \pi^-\pi^0$ , and  $D^- \rightarrow K_S^0 \pi^+\pi^-\pi^-$ , small non- $K_S^0$  contributions ( $< 0.5\%$  for each mode) can be used as tags and retained in the ST yields. The  $D^- \rightarrow K_S^0 K_S^0 \pi^-$  decays are kept as  $D^- \rightarrow K_S^0 \pi^+\pi^-\pi^-$  ST candidates ( $\sim 4\%$ ). Especially,

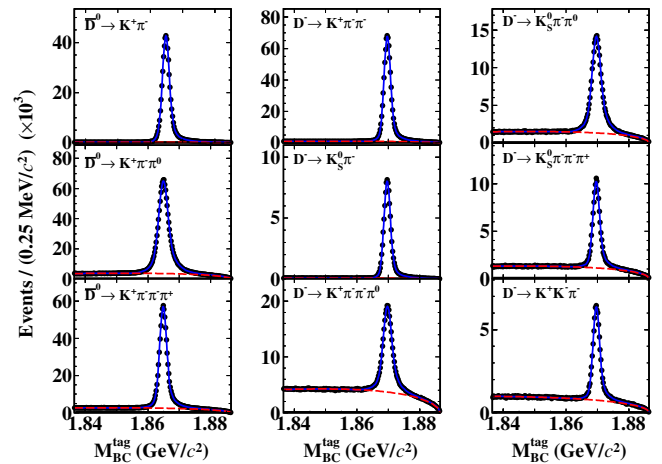


FIG. 1. Fits to the  $M_{\text{BC}}$  distributions of the ST  $\bar{D}^0$  (left column) and  $D^-$  (middle and right columns) candidates, where the points with error bars are data, the blue solid and red dashed curves are the fit results and the fitted backgrounds, respectively.

TABLE I. Requirements of  $\Delta E_{\text{sig}}$ , the fitted DT yields in the  $K_S^0$  signal region ( $N_{\text{DT}}^{\text{fit}}$ ), the fitted DT yields in the  $K_S^0$  sideband region ( $N_{K_S^0}^{\text{sid}}$ ), the net DT yields ( $N_{\text{DT}}^{\text{net}}$ ), the signal efficiencies ( $\epsilon_{\text{sig}}$ ), and the obtained BF's ( $\mathcal{B}_{\text{sig}}$ ) for various signal decays as well as comparisons with the world-average BF's ( $\mathcal{B}_{\text{PDG}}$ ). The first and second uncertainties for  $\mathcal{B}_{\text{sig}}$  are statistical and systematic, respectively, while the uncertainties for  $N_{\text{DT}}^{\text{net}}$  and  $\epsilon_{\text{sig}}$  are statistical only. The world-average BF of  $D^+ \rightarrow K^+ K^- \pi^+ \pi^0$  is obtained by summing over the contributions of  $D^+ \rightarrow \phi(\rightarrow K^+ K^-) \pi^+ \pi^0$  and  $D^+ \rightarrow K^+ K^- \pi^+ \pi^0|_{\text{non-}\phi}$ .

Signal mode	$\Delta E_{\text{sig}}$	$N_{\text{DT}}^{\text{fit}}$	$N_{K_S^0}^{\text{sid}}$	$N_{\text{DT}}^{\text{net}}$	$\epsilon_{\text{sig}}$ (%)	$\mathcal{B}_{\text{sig}} (\times 10^{-3})$	$\mathcal{B}_{\text{PDG}} (\times 10^{-3})$
$D^0 \rightarrow K^+ K^- \pi^0 \pi^0$	(-59, 40)	$132.1 \pm 13.9$	...	$132.1 \pm 13.9$	$8.20 \pm 0.07$	$0.69 \pm 0.07 \pm 0.04$	...
$D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$	(-22, 22)	$82.1 \pm 9.7$	$37.8 \pm 7.5$	$63.2 \pm 10.4$	$5.14 \pm 0.04$	$0.53 \pm 0.09 \pm 0.03$	$1.22 \pm 0.23$
$D^0 \rightarrow K_S^0 K^- \pi^+ \pi^0$	(-43, 32)	$278.8 \pm 18.8$	$166.1 \pm 15.1$	$195.8 \pm 20.3$	$6.38 \pm 0.06$	$1.32 \pm 0.14 \pm 0.07$	...
$D^0 \rightarrow K_S^0 K^+ \pi^- \pi^0$	(-44, 33)	$124.0 \pm 12.8$	$9.5^{+3.7}_{-3.1}$	$119.3 \pm 12.9$	$7.94 \pm 0.06$	$0.65 \pm 0.07 \pm 0.02$	...
$D^+ \rightarrow K^+ K^- \pi^+ \pi^0$	(-39, 30)	$1311.7 \pm 40.4$	...	$1311.7 \pm 40.4$	$12.72 \pm 0.08$	$6.62 \pm 0.20 \pm 0.25$	$26^{+9}_{-8}$
$D^+ \rightarrow K_S^0 K^+ \pi^0 \pi^0$	(-61, 44)	$35.9 \pm 7.1$	$3.8^{+2.8}_{-2.0}$	$34.0 \pm 7.2$	$3.77 \pm 0.02$	$0.58 \pm 0.12 \pm 0.04$	...
$D^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$	(-22, 21)	$505.0 \pm 24.5$	$74.2 \pm 10.3$	$467.9 \pm 25.0$	$13.24 \pm 0.08$	$2.27 \pm 0.12 \pm 0.06$	$2.38 \pm 0.17$
$D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$	(-21, 20)	$284.6 \pm 18.0$	$15.3^{+4.9}_{-4.2}$	$277.0 \pm 18.2$	$9.39 \pm 0.06$	$1.89 \pm 0.12 \pm 0.05$	$1.74 \pm 0.18$
$D^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^0$	(-46, 37)	$101.1 \pm 11.3$	$42.0 \pm 8.1$	$80.1 \pm 12.0$	$3.84 \pm 0.03$	$1.34 \pm 0.20 \pm 0.06$	...

the  $\bar{D}^0 \rightarrow K_S^0 K^+ \pi^-$  and  $\bar{D}^0 \rightarrow K_S^0 K^- \pi^+$  decays, which contribute  $\sim 2.6\%$  to the  $\bar{D}^0 \rightarrow K^+ \pi^- \pi^- \pi^+$  ST candidates, have been subtracted due to the strong-phase effects.

## VI. YIELDS OF DT EVENTS

In the recoiling sides against the tagged  $\bar{D}$  candidates, the signal  $D$  decays are selected by using the residual tracks that have not been used to reconstruct the tagged  $\bar{D}$  candidates. To suppress the  $K_S^0$  contribution in the individual mass spectra for the  $D^0 \rightarrow K^+ K^- \pi^0 \pi^0$ ,  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$ , and  $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$  decays, the  $\pi^+ \pi^-$  and  $\pi^0 \pi^0$  invariant masses are required to be outside (0.468, 0.528)  $\text{GeV}/c^2$  and (0.438, 0.538)  $\text{GeV}/c^2$ , respectively. To suppress the background from  $D^0 \rightarrow K^- \pi^+ \omega$  in the identification of the  $D^0 \rightarrow K_S^0 K^- \pi^+ \pi^0$  process, the  $K_S^0 \pi^0$  invariant mass is required to be outside (0.742, 0.822)  $\text{GeV}/c^2$ . These requirements correspond to at least five times the fitted mass resolution away from the fitted mean of the mass peak.

The signal  $D$  mesons are identified using the energy difference  $\Delta E_{\text{sig}}$  and the beam-constrained mass  $M_{\text{BC}}^{\text{sig}}$ , which are calculated with Eqs. (3) and (4) by substituting “tag” with “sig”. For each signal mode, if there are multiple candidates in an event, only the one with the smallest  $|\Delta E_{\text{sig}}|$  is kept. The signal decays are required to satisfy the mode-dependent  $\Delta E_{\text{sig}}$  requirements, as shown in the second column of Table I. To suppress incorrectly identified  $D\bar{D}$  candidates, the opening angle between the tagged  $\bar{D}$  and the signal  $D$  is required to be greater than  $160^\circ$ , resulting in a loss of (2–6)% of the signal and suppressing (8–55)% of the background.

Figure 2 shows the  $M_{\text{BC}}^{\text{tag}}$  versus  $M_{\text{BC}}^{\text{sig}}$  distribution of the accepted DT candidates in data. The signal events concentrate around  $M_{\text{BC}}^{\text{tag}} = M_{\text{BC}}^{\text{sig}} = M_D$ , where  $M_D$  is the

nominal  $D$  mass [9]. The events with correctly reconstructed  $D$  ( $\bar{D}$ ) and incorrectly reconstructed  $\bar{D}$  ( $D$ ), named BKGI, are spread along the lines around  $M_{\text{BC}}^{\text{tag}} = M_D$  or  $M_{\text{BC}}^{\text{sig}} = M_D$ . The events smeared along the diagonal, named BKGII, are mainly from the  $e^+ e^- \rightarrow q\bar{q}$  processes. The events with uncorrelated and incorrectly reconstructed  $D$  and  $\bar{D}$ , named BKGIII, disperse across the whole allowed kinematic region. Usually, the horizontal and vertical BKGI components are caused mainly due to particle misidentification(s), fake or missing  $\pi^0$ (s) in the signal and tag sides, respectively. For example, inclusive MC studies show that the largest source of the horizontal

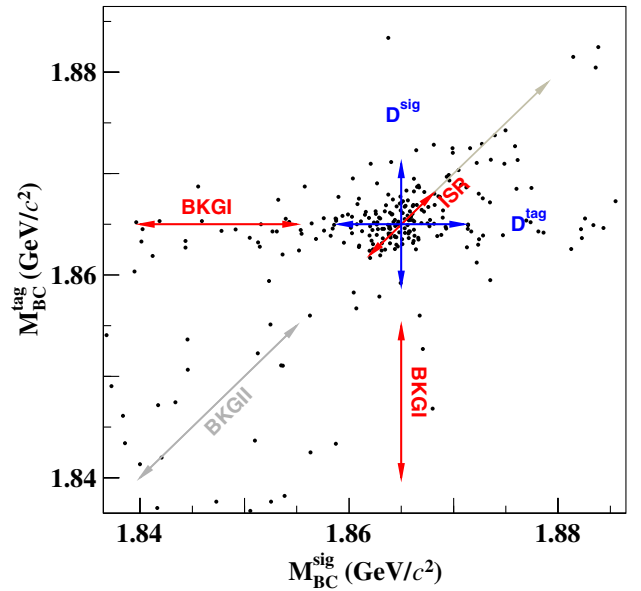


FIG. 2. The  $M_{\text{BC}}^{\text{tag}}$  versus  $M_{\text{BC}}^{\text{sig}}$  distribution of the accepted DT candidates of  $D^0 \rightarrow K^+ K^- \pi^0 \pi^0$  in data. Here, ISR denotes the signal spreading along the diagonal direction.

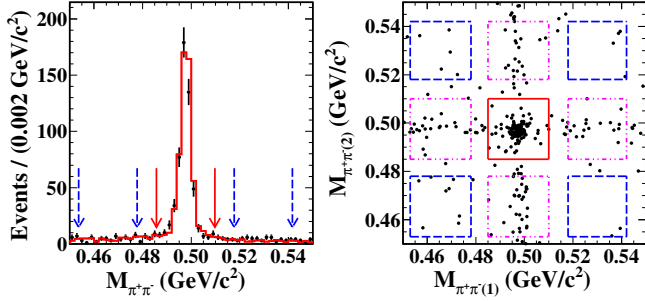


FIG. 3. (a) The  $\pi^+\pi^-$  invariant-mass distributions of the  $D^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$  candidate events of data (points with error bars) and inclusive MC sample (histogram). Pairs of the red solid (blue dashed) arrows denote the  $K_S^0$  signal (sideband) regions. (b) Distribution of  $M_{\pi^+\pi^-(1)}$  versus  $M_{\pi^+\pi^-(2)}$  for the  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$  candidate events in data. Red solid box denotes the 2D signal region. Pink dot-dashed (blue dashed) boxes indicate the 2D sideband 1 (2) regions.

BKGI for the  $D^0 \rightarrow K^+ K^- \pi^0 \pi^0$  signal decay is from  $D^0 \rightarrow \phi K_S^0 (\rightarrow \pi^0 \pi^0)$  with one fake  $\pi^0$ , which contribute  $\sim 23\%$  to the total horizontal BKGI.

For each signal  $D$  decay mode, the yield of DT events ( $N_{\text{DT}}^{\text{fit}}$ ) is obtained from a two-dimensional (2D) unbinned maximum-likelihood fit [34] on the  $M_{\text{BC}}^{\text{tag}}$  versus  $M_{\text{BC}}^{\text{sig}}$  distribution of the accepted candidates. In the fit, the probability density functions (PDFs) of signal, BKGI, BKGII, and BKGIII are constructed as

- (i) signal:  $a(x, y)$ ,
- (ii) BKGI:  $b(x) \cdot c_y(y; E_b, \xi_y) + b(y) \cdot c_x(x; E_b, \xi_x)$ ,
- (iii) BKGII:  $c_z(z; \sqrt{2}E_b, \xi_z) \cdot g(k)$ , and
- (iv) BKGIII:  $c_x(x; E_b, \xi_x) \cdot c_y(y; E_b, \xi_y)$ ,

respectively. Here,  $x = M_{\text{BC}}^{\text{sig}}$ ,  $y = M_{\text{BC}}^{\text{tag}}$ ,  $z = (x + y)/\sqrt{2}$ , and  $k = (x - y)/\sqrt{2}$ . The PDFs of signal  $a(x, y)$ ,  $b(x)$ , and

$b(y)$  are described by the corresponding MC-simulated shapes.  $c_f(f; E_{\text{end}}, \xi_f)$  is an ARGUS function [33] defined above, where  $f$  denotes  $x$ ,  $y$ , or  $z$ ;  $E_b$  is fixed at 1.8865 GeV.  $g(k)$  is a Gaussian function with mean of zero and standard deviation parametrized by  $\sigma_k = \sigma_0 \cdot (\sqrt{2}E_b - z)^p$ , where  $\sigma_0$  and  $p$  are fit parameters.

Combinatorial  $\pi^+\pi^-$  pairs from the decays  $D^0 \rightarrow K_S^0 2(\pi^+\pi^-)$  [and  $D^0 \rightarrow 3(\pi^+\pi^-)$ ],  $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^- \pi^0$ ,  $D^0 \rightarrow K^+ \pi^+ \pi^- \pi^- \pi^0$ ,  $D^+ \rightarrow K^- \pi^+ \pi^+ \pi^+ \pi^-$ ,  $D^+ \rightarrow K^+ 2(\pi^+\pi^-)$ ,  $D^+ \rightarrow K^+ \pi^+ \pi^- \pi^0 \pi^0$ ,  $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^- \pi^0$  [and  $D^+ \rightarrow 2(\pi^+\pi^-) \pi^+ \pi^0$ ] may also satisfy the  $K_S^0$  selection criteria and form peaking backgrounds around  $M_D$  in the  $M_{\text{BC}}^{\text{sig}}$  distributions for  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$ ,  $D^0 \rightarrow K_S^0 K^- \pi^+ \pi^0$ ,  $D^0 \rightarrow K_S^0 K^+ \pi^- \pi^0$ ,  $D^+ \rightarrow K_S^0 K^+ \pi^0 \pi^0$ ,  $D^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$ ,  $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ , and  $D^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^0$ , respectively. This kind of peaking background is estimated by selecting events in the  $K_S^0$  sideband region of  $(0.454, 0.478) \cup (0.518, 0.542)$  GeV/ $c^2$ . For  $D^0 \rightarrow K_S^0 K^- \pi^+ \pi^0$ ,  $D^0 \rightarrow K_S^0 K^+ \pi^- \pi^0$ ,  $D^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$ ,  $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ , and  $D^+ \rightarrow K_S^0 K^+ \pi^0 \pi^0$  decays, one-dimensional (1D) signal and sideband regions are used. For  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$  and  $D^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^0$  decays, 2D signal and sideband regions are used. The 2D  $K_S^0$  signal region is defined as the square region with both  $\pi^+\pi^-$  combinations lying in the  $K_S^0$  signal regions. The 2D  $K_S^0$  sideband 1 (2) regions are defined as the square regions with 1 (2)  $\pi^+\pi^-$  combination(s) located in the 1D  $K_S^0$  sideband regions and the rest in the 1D  $K_S^0$  signal region. Figure 3 shows 1D and 2D  $\pi^+\pi^-$  invariant-mass distributions as well as the  $K_S^0$  signal and sideband regions.

For the signal decays involving  $K_S^0$  meson(s) in the final states, the net yields of DT events are calculated by subtracting the sideband contribution from the DT fitted yield by

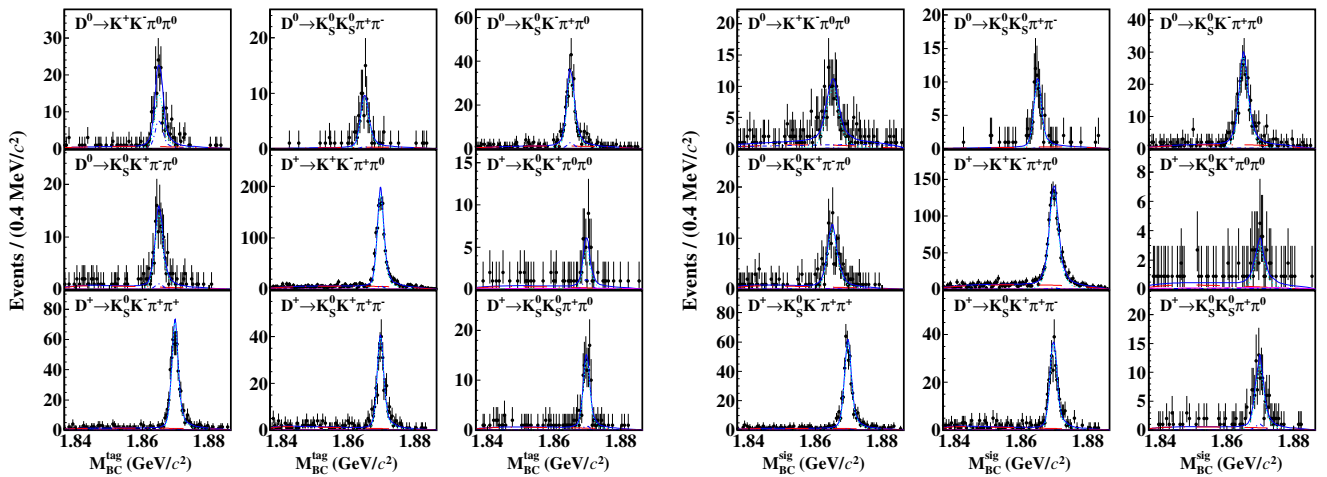


FIG. 4. Projections on the  $M_{\text{BC}}^{\text{tag}}$  and  $M_{\text{BC}}^{\text{sig}}$  distributions of the 2D fits to the DT candidate events with all  $\bar{D}^0$  or  $D^-$  tags. Data are shown as points with error bars. Blue solid, light blue dotted, blue dot-dashed, red dot-long-dashed, and pink long-dashed curves denote the overall fit results, signal, BKGI, BKGII, and BKGIII components (see text), respectively.

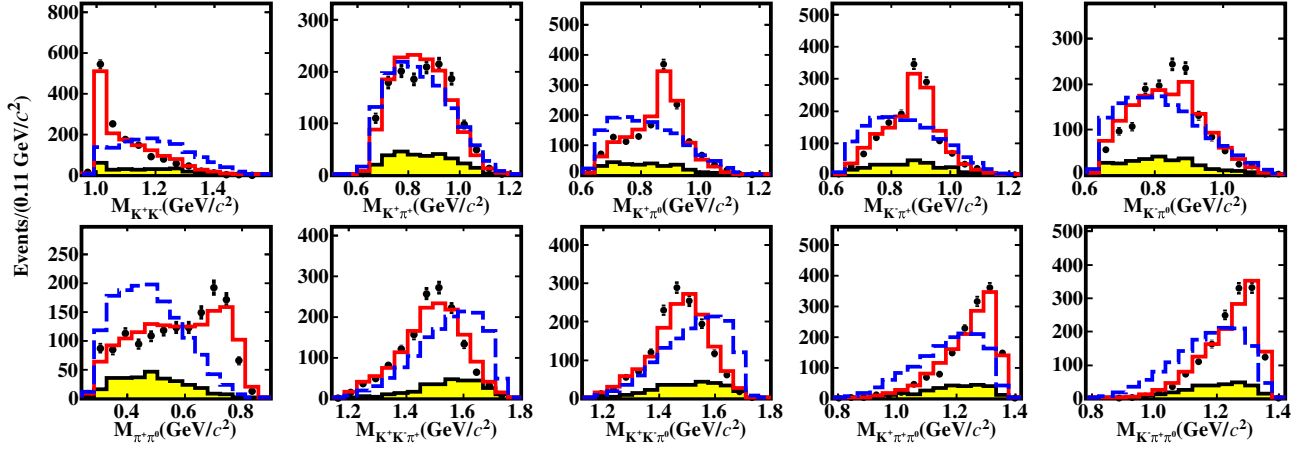


FIG. 5. The invariant mass distributions of two or three-body particle combinations of  $D^+ \rightarrow K^+K^-\pi^+\pi^0$  candidate events for data and MC simulations. Data are shown as points with error bars. Yellow hatched histograms are the backgrounds estimated from the inclusive MC sample. Red solid histograms are the mixed signal MC samples plus MC-simulated backgrounds. Blue dashed histograms are the PHSP signal MC samples plus MC-simulated backgrounds. Events have been required to be within  $|M_{\text{BC}}^{\text{tag}(\text{sig})} - M_D| < 0.006 \text{ GeV}/c^2$ .

$$N_{\text{DT}}^{\text{net}} = N_{\text{DT}}^{\text{fit}} - \sum_i^N \left[ \left( -\frac{1}{2} \right)^{i+1} N_{\text{side}_i}^{\text{fit}} \right] = N_{\text{DT}}^{\text{fit}} - \frac{1}{2} N_{K_S^0}^{\text{sid}}. \quad (5)$$

Here,  $N = 1$  for the decays with one  $K_S^0$  meson while  $N = 2$  for the decays with two  $K_S^0$  mesons. The combinatorial  $\pi^+\pi^-$  backgrounds are assumed to be uniformly distributed and double-counting is avoided by subtracting (2) yields from (1) yields appropriately.  $N_{\text{DT}}^{\text{fit}}$  and  $N_{\text{side}_i}^{\text{fit}}$  are the fitted  $D$  yields in the 1D or 2D signal region and sideband  $i$  region, respectively. For the other signal decays, the net yields of DT events are  $N_{\text{DT}}^{\text{fit}}$ . Figure 4 shows the  $M_{\text{BC}}^{\text{tag}}$  and  $M_{\text{BC}}^{\text{sig}}$  projections of the 2D fits to data. From these 2D fits, we obtain the DT yields for individual signal decays as shown in Table I.

For each signal decay mode, the statistical significance is calculated according to  $\sqrt{-2\ln(\mathcal{L}_0/\mathcal{L}_{\text{max}})}$ , where  $\mathcal{L}_{\text{max}}$  and  $\mathcal{L}_0$  are the maximum likelihoods of the fits with and without involving the signal component, respectively. The effect of combinatorial  $\pi^+\pi^-$  backgrounds in the  $K_S^0$ -signal regions has been considered for the decays involving a  $K_S^0$ . The statistical significance for each signal decay is found to be greater than  $8\sigma$ .

## VII. RESULTS

Each of the  $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$ ,  $D^+ \rightarrow K^+K^-\pi^+\pi^0$ ,  $D^+ \rightarrow K_S^0K^-\pi^+\pi^+$ , and  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$  decays is modeled by the corresponding mixed signal MC samples, in which the dominant decay modes containing resonances of  $K^*(892)$ ,  $\rho(770)$ , and  $\phi$  are mixed with the phase space (PHSP) signal MC samples. The mixing ratios are determined by examining the corresponding invariant mass and momentum spectra. The other decays, which are limited in statistics, are generated with the PHSP generator. The

momentum and the polar angle distributions of the daughter particles and the invariant masses of each two- and three-body particle combinations of the data agree with those of the MC simulations. As an example, Fig. 5 shows the invariant mass distributions of two- or three-body particle combinations of  $D^+ \rightarrow K^+K^-\pi^+\pi^0$  candidate events for data and MC simulations.

The measured values of  $N_{\text{DT}}^{\text{net}}$ ,  $\epsilon_{\text{sig}}$ , and the obtained BFs are summarized in Table I. The current world-average values are also given for comparison. The signal efficiencies have been corrected by the necessary data-MC differences in the selection efficiencies of  $K^\pm$  and  $\pi^\pm$  tracking and PID procedures and the  $\pi^0$  reconstruction. These efficiencies include the BFs of the  $K_S^0$  and  $\pi^0$  decays, obtained by removing  $K_S^0$  sideband contribution. The difference of efficiencies with and without removing  $K_S^0$  sideband contribution is (2–3)%.

The efficiencies for  $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$  and  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$  before  $K_S^0$  ( $\omega$ ) rejection are  $(8.23 \pm 0.07)\%$  and  $(10.89 \pm 0.07)\%$ , respectively. Our nominal efficiency for  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$  ( $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$ ) is lower than that of  $D^+ \rightarrow K_S^0K^-\pi^+\pi^+$  ( $D^0 \rightarrow K_S^0K^+\pi^-\pi^0$ ) due to different intermediate resonant states and further  $K_S^0$  ( $\omega$ ) rejection in the  $\pi^+\pi^-$  ( $K_S^0\pi^0$ ) mass spectrum.

## VIII. SYSTEMATIC UNCERTAINTIES

The systematic uncertainties are estimated relative to the measured BFs and are discussed below. In BF determinations using Eq. (1), all uncertainties associated with the selection of tagged  $\bar{D}$  canceled in the ratio. The systematic uncertainties in the total yields of ST  $\bar{D}$  mesons related to the  $M_{\text{BC}}$  fits to the ST  $\bar{D}$  candidates, were previously



estimated to be 0.5% for both neutral and charged  $\bar{D}$  [22–24].

The tracking and PID efficiencies for  $K^\pm$  or  $\pi^\pm$ ,  $\epsilon_{K\text{or}\pi}^{\text{tracking(PID)}}[\text{data}]$  and  $\epsilon_{K\text{or}\pi}^{\text{tracking(PID)}}[\text{MC}]$ , are investigated using DT  $D\bar{D}$  hadronic events. The averaged ratios between data and MC efficiencies ( $f_{K\text{or}\pi}^{\text{tracking(PID)}} = \epsilon_{K\text{or}\pi}^{\text{tracking(PID)}}[\text{data}] / \epsilon_{K\text{or}\pi}^{\text{tracking(PID)}}[\text{MC}]$ ) of tracking (PID) for  $K^\pm$  or  $\pi^\pm$  are weighted by the corresponding momentum spectra of signal MC events, giving  $f_K^{\text{tracking}}$  to be 1.022 – 1.031 and  $f_\pi^{\text{tracking}}$  to be close to unity. After correcting the MC efficiencies by  $f_K^{\text{tracking}}$ , the residual uncertainties of  $f_{K\text{or}\pi}^{\text{tracking}}$  are assigned as the systematic uncertainties of tracking efficiencies, which are (0.4–0.7)% per  $K^\pm$  and (0.2–0.3)% per  $\pi^\pm$ .  $f_K^{\text{PID}}$  and  $f_\pi^{\text{PID}}$  are all close to unity and their individual uncertainties, (0.2–0.3)%, are taken as the associated systematic uncertainties per  $K^\pm$  or  $\pi^\pm$ .

The systematic error related to the uncertainty in the  $K_S^0$  reconstruction efficiency is estimated from measurements of  $J/\psi \rightarrow K^*(892)^\mp K^\pm$  and  $J/\psi \rightarrow \phi K_S^0 K^\pm \pi^\mp$  control samples [35] and found to be 1.6% per  $K_S^0$ . The systematic uncertainty of  $\pi^0$  reconstruction efficiency is assigned as (0.7–0.8)% per  $\pi^0$  from a study of DT  $D\bar{D}$  hadronic decays of  $\bar{D}^0 \rightarrow K^+ \pi^- \pi^0$  and  $\bar{D}^0 \rightarrow K_S^0 \pi^0$  decays tagged by either  $D^0 \rightarrow K^- \pi^+$  or  $D^0 \rightarrow K^- \pi^+ \pi^+$  [22,23].

The systematic uncertainty in the 2D fit to the  $M_{\text{BC}}^{\text{tag}}$  versus  $M_{\text{BC}}^{\text{sig}}$  distribution is examined via the repeated measurements in which the signal shape and the endpoint of the ARGUS function ( $\pm 0.2$  MeV/ $c^2$ ) are varied. Quadratically summing the changes of the BFs for these two sources yields the corresponding systematic uncertainties.

The systematic uncertainty due to the  $\Delta E_{\text{sig}}$  requirement is assigned to be 0.3%, which corresponds to the largest efficiency difference with and without smearing the data-MC Gaussian resolution of  $\Delta E_{\text{sig}}$  for signal MC events. Here, the smeared Gaussian parameters are obtained by using the samples of DT events  $D^0 \rightarrow K_S^0 \pi^0$ ,  $D^0 \rightarrow K^- \pi^+ \pi^0$ ,  $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ , and  $D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$  versus the same  $\bar{D}$  tags in our nominal analysis. The systematic uncertainties due to  $K_S^0$  sideband choice and  $K_S^0$  rejection mass window are assigned by examining the changes of the BFs via varying nominal  $K_S^0$  sideband and corresponding rejection window by  $\pm 5$  MeV/ $c^2$ .

For the decays whose efficiencies are estimated with mixed signal MC events, the systematic uncertainty in the MC modeling is determined by comparing the signal efficiency when changing the percentage of MC sample components. For the decays whose efficiencies are estimated with PHSP-distributed signal MC events, the uncertainties are assigned as the change of the signal efficiency after adding the possible decays containing  $K^*(892)$  or  $\rho(770)$ . The imperfect simulations of the momentum and  $\cos\theta$  distributions of charged particles are considered as a source of systematic uncertainty. The signal efficiencies are reweighted by those distributions in data with background subtracted. The largest change of the reweighted to nominal efficiencies, 0.9%, is assigned as the corresponding systematic uncertainty.

The measurements of the BFs of the neutral  $D$  decays are affected by quantum correlation effect. For each neutral  $D$  decay, the  $CP$ -even component is estimated by the  $CP$ -even tag  $D^0 \rightarrow K^+ K^-$  and the  $CP$ -odd tag  $D^0 \rightarrow K_S^0 \pi^0$ . Using the same method as described in Ref. [36] and the

TABLE II. Systematic uncertainties (%) in the measurements of the BFs of the signal decays (1)  $D^0 \rightarrow K^+ K^- \pi^0 \pi^0$ , (2)  $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$ , (3)  $D^0 \rightarrow K_S^0 K^- \pi^+ \pi^0$ , (4)  $D^0 \rightarrow K_S^0 K^+ \pi^- \pi^0$ , (5)  $D^+ \rightarrow K^+ K^- \pi^+ \pi^0$ , (6)  $D^+ \rightarrow K_S^0 K^+ \pi^0 \pi^0$ , (7)  $D^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$ , (8)  $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ , and (9)  $D^+ \rightarrow K_S^0 K_S^0 \pi^+ \pi^0$ .

Source/Signal decay	1	2	3	4	5	6	7	8	9
$N_{\text{ST}}^{\text{tot}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$(K/\pi)^\pm$ tracking	1.0	0.6	0.9	0.9	1.6	0.4	1.1	1.2	0.3
$(K/\pi)^\pm$ PID	0.4	0.4	0.6	0.6	1.0	0.2	0.6	0.7	0.2
$K_S^0$ reconstruction	...	3.2	1.6	1.6	...	1.6	1.6	1.6	3.2
$\pi^0$ reconstruction	1.6	...	0.7	0.7	0.8	1.6	...	...	0.7
$\Delta E_{\text{sig}}$ requirement	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
$K_S^0$ rejection	4.2	2.4	...	...	...	4.2	...	0.8	...
$K_S^0$ sideband	...	0.2	1.1	0.2	...	1.3	0.1	0.1	0.2
Quoted BFs	0.0	0.1	0.1	0.1	0.0	0.1	0.1	0.1	0.1
MC statistics	0.8	0.6	0.7	0.6	0.5	0.4	0.4	0.5	0.6
MC modeling	1.3	1.0	0.5	0.7	2.1	1.4	0.5	0.7	0.5
Imperfect simulation	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
$D\bar{D}$ opening angle	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
2D fit	1.3	2.8	3.1	1.5	1.9	2.7	0.5	0.6	3.0
Quantum correlation effect	1.6	2.8	3.4	1.1	...	...	...	...	...
Total	5.4	5.9	5.4	3.2	3.8	6.0	2.5	2.8	4.7

necessary parameters quoted from Refs. [37–39], we find the correction factors to account for the quantum correlation effect on the measured BFs are  $(98.3_{-1.1}^{+1.6})\%$ ,  $(98.1_{-1.7}^{+2.8})\%$ ,  $(95.9_{-2.7}^{+3.4})\%$ , and  $(98.4_{-1.0}^{+1.1})\%$  for  $D^0 \rightarrow K^+K^-\pi^0\pi^0$ ,  $D^0 \rightarrow K_S^0K_S^0\pi^+\pi^-$ ,  $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$ , and  $D^0 \rightarrow K_S^0K^+\pi^-\pi^0$ , respectively. After correcting the signal efficiencies by the individual factors, the residual uncertainties are assigned as systematic uncertainties.

The uncertainties due to the limited MC statistics for various signal decays, (0.4–0.8)%, are taken into account as a systematic uncertainty. The uncertainties of the quoted BFs of the  $K_S^0 \rightarrow \pi^+\pi^-$  and  $\pi^0 \rightarrow \gamma\gamma$  decays are 0.07% and 0.03%, respectively [9].

The efficiencies of  $D\bar{D}$  opening angle requirement is studied by using the DT events of  $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ ,  $D^0 \rightarrow K^-\pi^+\pi^0\pi^0$ , and  $D^0 \rightarrow K^-\pi^+\pi^0$  tagged by the same tag modes in our nominal analysis. The largest difference of the accepted efficiencies between data and MC simulations, 0.4%, is assigned as the associated systematic uncertainty.

Table II summarizes the systematic uncertainties in the BF measurements. For each signal decay, the total systematic uncertainty is obtained by adding the above effects in quadrature to be (2.5–6.0)% for various signal decay modes.

## IX. SUMMARY

In summary, by analyzing a data sample obtained in  $e^+e^-$  collisions at  $\sqrt{s} = 3.773$  GeV with the BESIII detector and corresponding to an integrated luminosity of  $2.93 \text{ fb}^{-1}$ , we obtained the first direct measurements of the absolute BFs of nine  $D^{0(+)} \rightarrow K\bar{K}\pi\pi$  decays containing  $K_S^0$  or  $\pi^0$  mesons. The  $D^0 \rightarrow K^+K^-\pi^0\pi^0$ ,  $D^0 \rightarrow K_S^0K^-\pi^+\pi^0$ ,  $D^0 \rightarrow K_S^0K^+\pi^-\pi^0$ ,  $D^+ \rightarrow K_S^0K^+\pi^0\pi^0$ , and  $D^+ \rightarrow K_S^0K_S^0\pi^+\pi^0$  decays are observed for the first time. Compared to the world-average values, the BFs of the  $D^0 \rightarrow K_S^0K_S^0\pi^+\pi^-$ ,  $D^+ \rightarrow K^+K^-\pi^+\pi^0$ ,  $D^+ \rightarrow K_S^0K^-\pi^+\pi^+$ , and  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$  decays are measured with improved precision. Our BFs of  $D^+ \rightarrow K_S^0K^-\pi^+\pi^+$  and  $D^+ \rightarrow K_S^0K^+\pi^+\pi^-$  are in agreement with individual world averages within  $1\sigma$  while our BFs of  $D^0 \rightarrow K_S^0K_S^0\pi^+\pi^-$  and  $D^+ \rightarrow K^+K^-\pi^+\pi^0$  deviate with individual world averages

by  $2.3\sigma$  and  $2.8\sigma$ , respectively. The precision of the BF of  $D^+ \rightarrow K^+K^-\pi^+\pi^0$  is improved by a factor of about seven. Future amplitude analyses of all these  $D^{0(+)} \rightarrow K\bar{K}\pi\pi$  decays with larger data samples foreseen at BESIII [40], Belle II [41], and LHCb [42] will supply rich information of the two-body decay modes containing scalar, vector, axial and tensor mesons, thereby benefiting the understanding of quark SU(3)-flavor symmetry.

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